Discrete Surfaces and Manifolds:

A Potential tool to Image Processing

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Introduction

- Purpose: to introduce a potential research area related to Discrete Math, Image Processing, and Algorithm Design.
- I have worked in this area since 1985, and I will focus on my research work in this presentation.

Topics of Discussion

- Why we need digital/discrete surfaces
- What is a digital surface or manifold
- How many types of digital surface points in 3D
- Discrete manifolds
- Gradually varied surfaces and discrete fitting
- Algorithms and applications

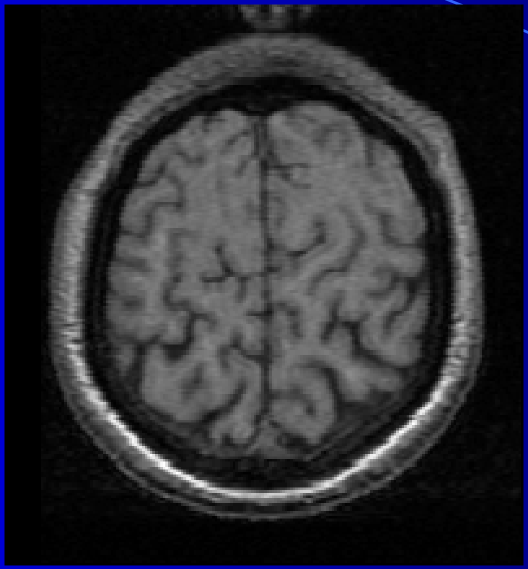
Why Digital/discrete Surfaces (1)

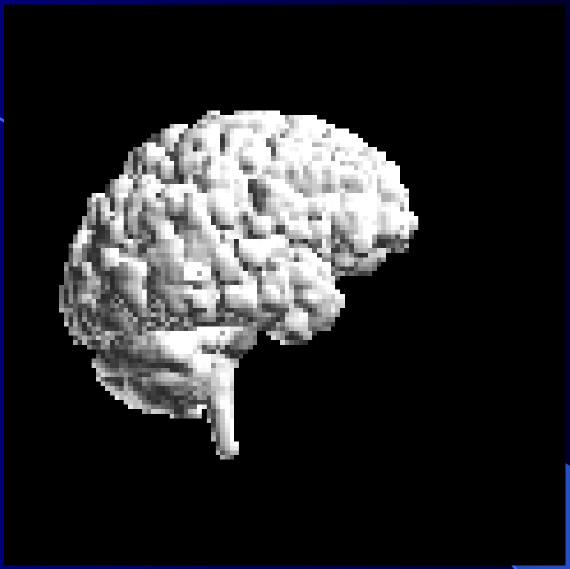
- Edge detection and 3D object tracking.

  Medical image examples:

  MR Brain images

- The result of the extraction will be digital surface.





Example:

MR Slice Image and segmented or tracked 3D brain

(Grégoire Malandain)

Why Digital/discrete Surfaces (2)

- A 2D gray scale image is a digital surface with or without continuity.

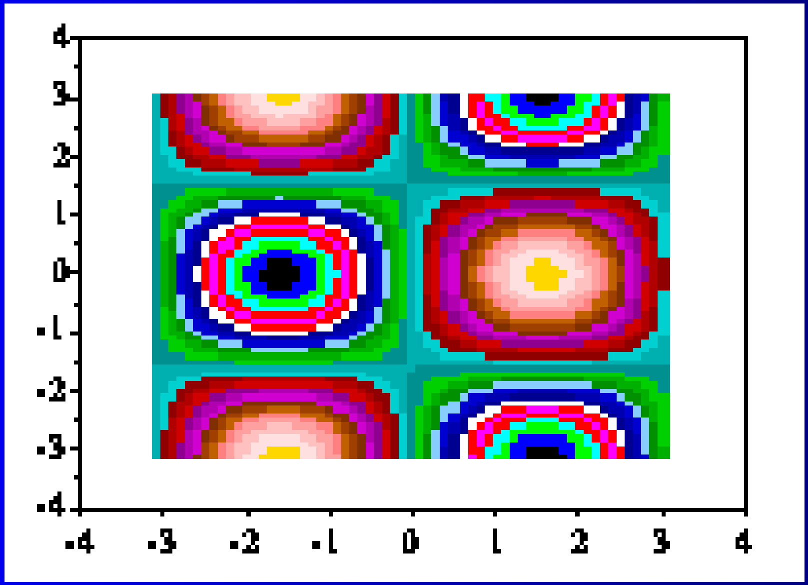
  Examples

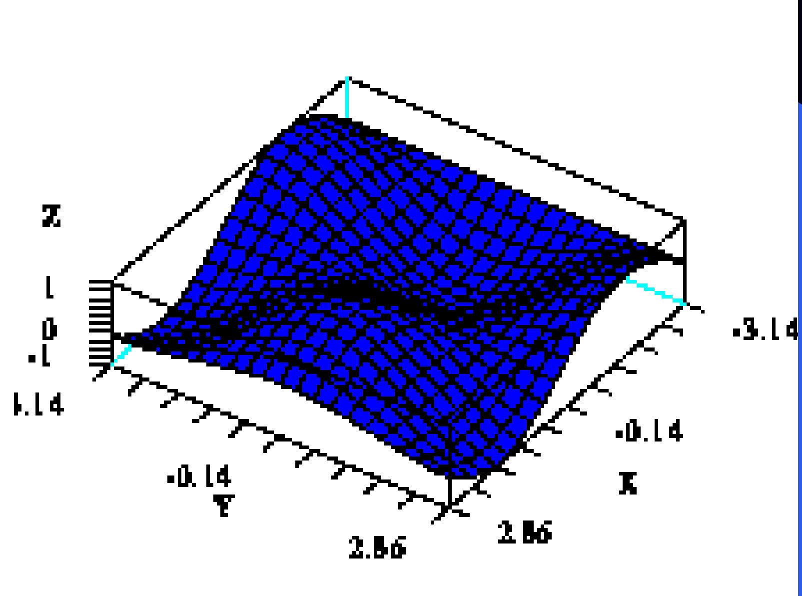
- An object in the image is often appeared to be a “continuously looking” segment.

  Or an object would be a “continuous” digital surface on a segmented region.

Example:

Gray Scale Image vs. Digital Surface





Questions

- How to design fast tracking?→
- What is a digital surface? →
- What type of geometrical/topological properties of digital surfaces can be used in tracking →
- How to describe a “continuous part” in the digital function. →

Why Digital/discrete Surfaces(3)

- Based on above two kinds of examples,

  We must know what is a "Digital Surface"

- This research relates discrete math, algorithms, and image processing

The Digital Surface and Manifold(1)

Definition of 3D digital surfaces:

- Artzy, Frieder, and Herman: *A digital surface is the boundary of a 3D digital object*. (Intuitive)

The Digital Surface and Manifold (2)

- Morgenthaler and Rosenfeld: *A digital surface is the set of surface points each of which has two adjacent components not in the surface in its neighborhood.* (Set-theoretic)
- Chen and Zhang: *A digital surface is formed by moving of a line-segment*. (Dynamic & recursive)

The Digital Surface and Manifold (3)

Basic Concepts:

- A point is 0-cell, a line segment is 1-cell, etc.
- An (i+1)-cell can be formed by two disjoint i-cells that are parallel. Or,
- An i-cell and its parallel move form an (i+1)-cell.

The Digital Surface and Manifold (4)

Definition of digital manifolds (Chen and Zhang, 1993): A connected subset S in digital space Σ is an i_D digital manifold if (give example for i=2):

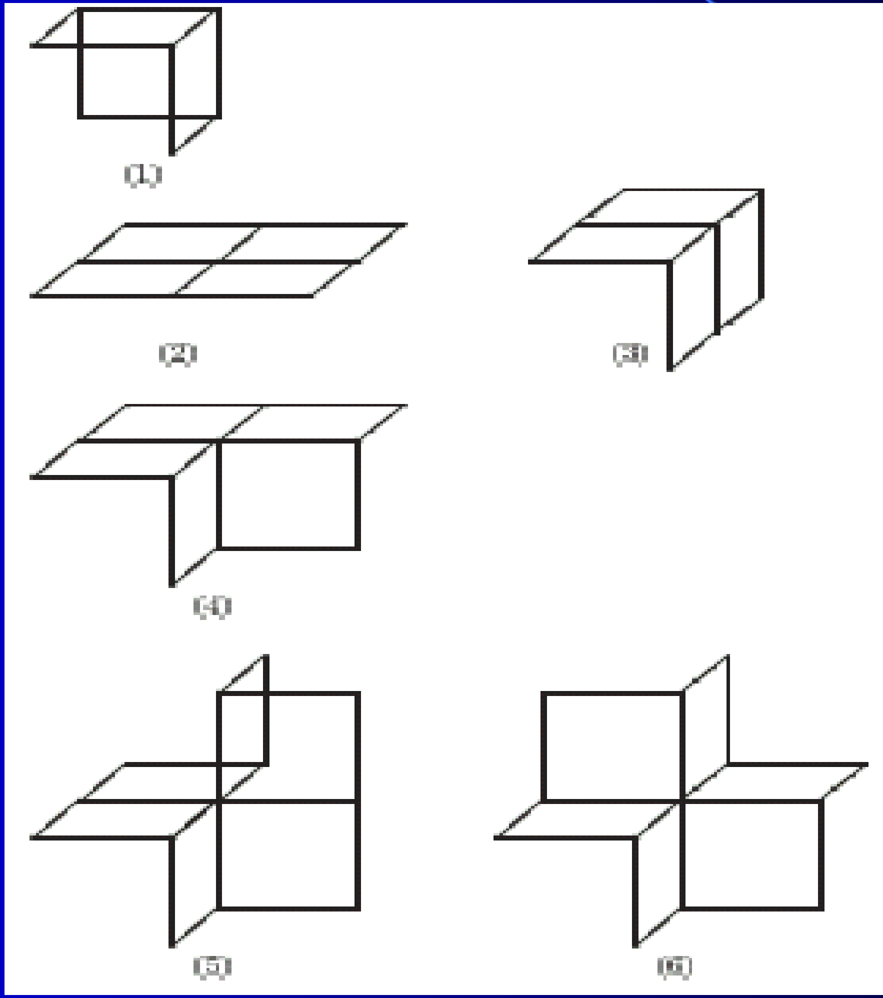
2) *Any two i-cells are (i-1)-connected in S,*
3) *Every (i-1)-cell in S has only one or two parallel-moves in S, and.*
4) *S does not contain any (i+1)-cell.*

Classification: Digital Surface Points in 3D

Chen *et al* obtained:

- **Theorem**: *The Morgenthaler-Rosenfeld's surface is equivalent to the surface defined by Chen and Zhang (in direct adjacency).*
- **Theorem:** *There are exactly 6 types of digital surface points in 3D (in direct adjacency).*

Classification(continue):
6 types of digital surface Points



Discrete Manifolds

- In most of cases, we are dealing with digital objects (grid spaces).
- Meshing in graphics deals with discrete objects.
- How to define discrete manifolds?

Similar to define digital manifolds, we can recursively define discrete manifolds

The Gradually Varied Surface: a Special Discrete Surface

- Gradual variation: let f: D→{1, 2,…,n}, if a and b are adjacent in D implies |f(a)- f(b)| ≤ 1, point (a,f(a)) and (b,f(b)) are said to be gradually varied.
- A 2D function (surface) is said to be gradually varied if every adjacent pair are gradually varied.

The Gradually Varied Surface (Continue)

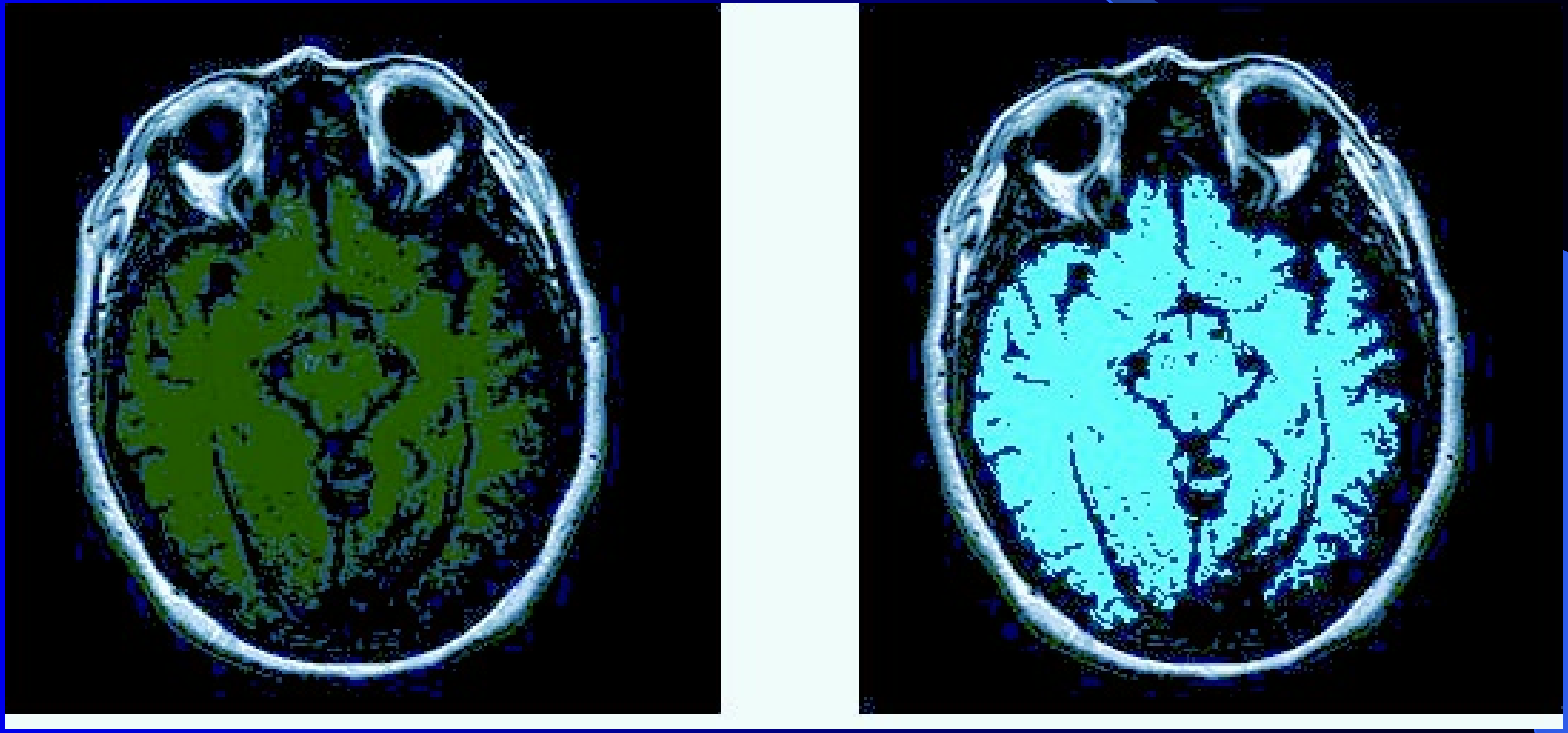
Remarks:

- This concept was called ``discretely continuous'' by Rosenfeld (1986) and ``roughly continuous'' by Pawlak (1995).
- A gradually varied function can be represented by lambda-connectedness introduced by Chen (1985).

Real Problems: Image Segmentation

- (Gray scale) image segmentation is to find all gradually varied components in an image. (Strong requirement, use split-and-merge technique)
- (Gray scale) image segmentation is to find all connected components in which for any pair of points, there is a gradually varied path to link them. (Weak requirement, use breadth-first-search technique) *Example*

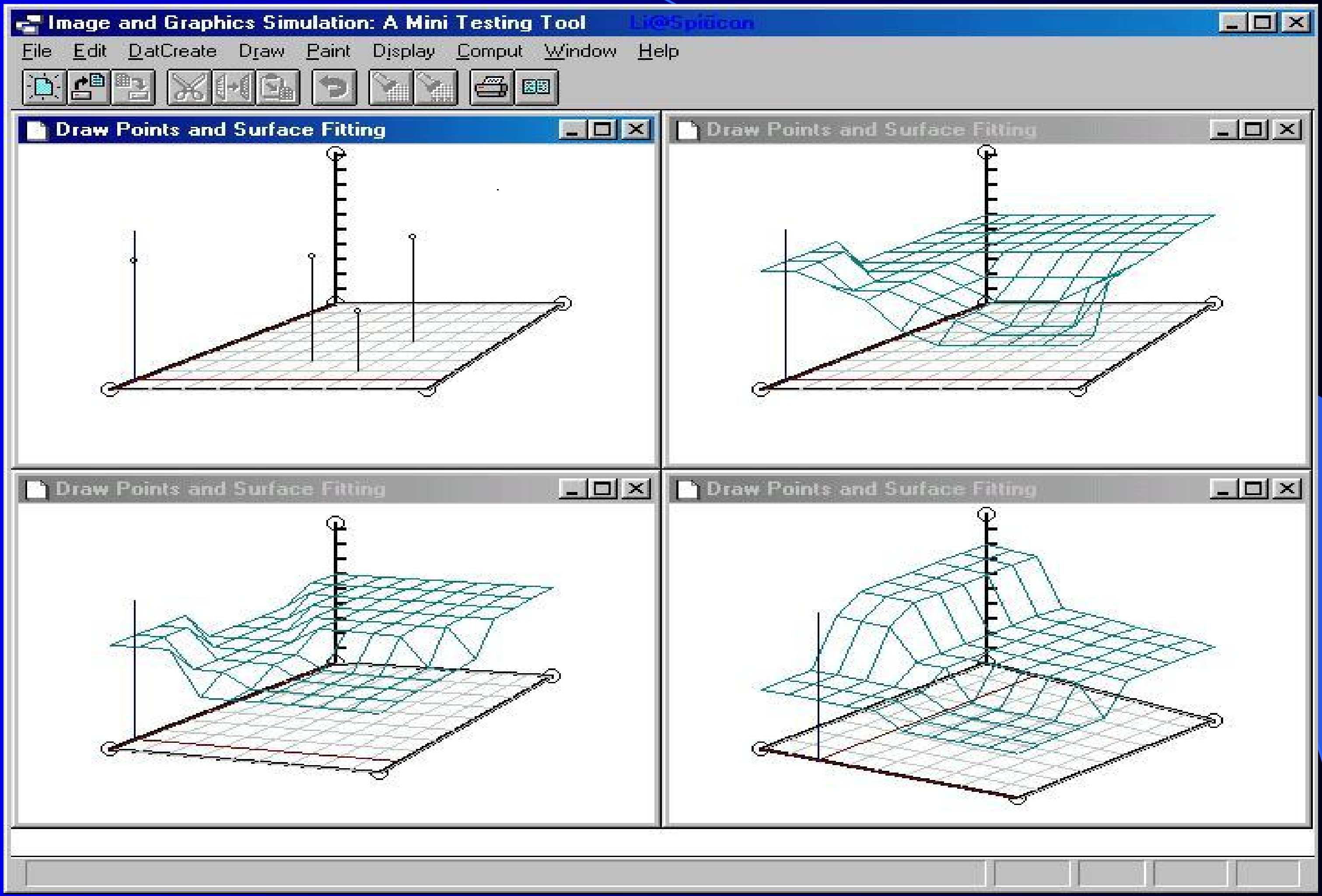
Example:
lambda-connected Segmentation



Real Problems: Discrete Surface Fitting

- Given J⊆D, and f: J→{1,2,…n} decide if there is a F: D→{1,2,…,n} such that F is gradually varied where f(x)=F(x), x in J.
- Theorem (Chen, 1989) the necessary and sufficient condition for the existence of a gradually varied extension F is: for all x,y in J, $d(x,y) \geq |f(x)-f(y)|$, where d is the distance between x and y in D.

Example: GVS fitting



Graph Immersion

Li Chen, Gradually varied surfaces and gradually varied functions, manuscript

Li Chen, Discrete Surfaces and Manifolds, SPC, 2004 . Chapter 8

Definition 2.1. *Let D_1 and D_2 be two discrete manifolds and $f : D_1 \to D_2$ be a mapping. f is said to be an immersion from D_1 to D_2 or a gradually varied operator if x and y are adjacent in D_1 implying $f(x) = f(y)$, or $f(x)$, $f(y)$ are adjacent in D_2.*

If $D_2 = \Sigma_m$, then f is called a gradually varied surface. An immersion f is said to be an embedding if f is a one-to-one mapping.

In fact, D_1 and D_2 can be two simple graphs in the above definition. (See [11] for concepts of graph theory.) In this case, we know a famous NP-complete problem [12] , the subgraph isomorphism problem, is related to the gradually varied operator.

Not Every Pair of D, D' have GV Extension

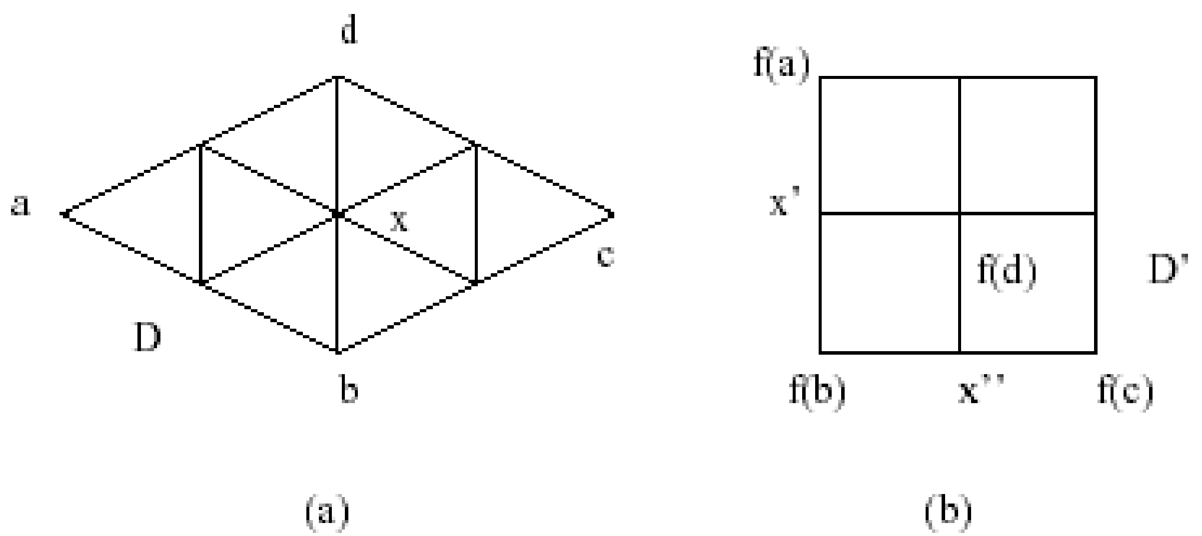


Fig. 3.1. The example that shows no gradually varied extension.

Normally Immersion/GV Mapping

Definition 3.1. *Let J be a subset of D and f_J be a mapping $f_J : J \to D'$, which satisfies:*

$$\forall p, q \in J[d(p,q) \geq d(f_J(p), f_J(q))]. \tag{3.1}$$

If there exists an extension f of f_J such that $f : D \to D'$ is a gradually varied mapping, then we say $< J, f_J >$ is immersion-extendable. If every $< J, f_J >$ satisfying (3.1) is immersion-extendable, then we say that D can normally immerse into D'.

The Main Results

We know that Theorem 2.1 states that any digital manifold can normally immerse to Σ_1 or a chain. We can show a more general theorem in the following:

Theorem 3.1. *Any graph D (or digital manifold) can normally immerse an arbitrary tree T.*

Corollary 3.1. *Any graph/digital manifold can normally immerse into an arbitrary forest.*

Theorem 3.2. *Any graph/digital manifold D can normally immerse into i_Σ_m.*

Problems in Algorithms and Applications

- 3D tracking and medical image processing, fast 3D chain code algorithms, 3D rendering.
- Tracking vs. Segmentation/Decision for single object, average time complexity analysis.
- Gradually varied segmentation using divide-and-conquer (split-and-merge) vs. Typical statistical method, how to deal with noise in gradually varied segmentation.

Problems in Algorithms and applications(continue)

- Gradually connected segmentation using breadth-first-search is similar to typical region-growing method.
- Fast gradually varied fitting algorithm development in the case of Jordan-separable-domain.
- Gradually varied fitting vs. numerical fitting

References

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- Please contact me at *lchen@udc.edu* if you are interested in my research.